BNL–NT–99/8
TAUP 2611–99
hep-ph/9912216

Scale Anomaly and "Soft" Pomeron in QCD

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Abstract

We propose a new non–perturbative approach to hadronic interactions at high energies and small momentum transfer, which is based on the scale anomaly of QCD and emphasizes the rôle of semi–classical vacuum fields. We find that the hadron scattering amplitudes exhibit Regge behavior and evaluate the intercept $\alpha(0)$ of the corresponding trajectory. Both the intercept and the scale for the slope of the trajectory appear to be determined by the energy density of non–perturbative QCD vacuum (the gluon condensate). Numerically, we find $\Delta \equiv \alpha(0) - 1 = 0.08 \div 0.1$, consistent with the values ascribed phenomenologically to the "soft" Pomeron. For arbitrary numbers of colors N_c and flavors N_f, Δ is found to be proportional to $(N_f/N_c)^2$; however, in the large N_c (N_f fixed) limit, $\Delta \sim N_c^0$.

Understanding the behavior of QCD at high energies and small momentum transfer is still a challenging and unsolved problem. In the framework of perturbation theory, a systematic approach was developed by Balitsky, Fadin, Kuraev and Lipatov [1], who demonstrated that the "leading log" terms in the scattering amplitude of type $(g^2\ ln\ s)^n$ (where g is the strong coupling) can be re-summed, giving rise to the so–called "hard" Pomeron. Diagramatically, BFKL equation describes the $t-$channel exchange of "gluonic ladder" (see Fig.1a) – a concept familiar from the old–fashioned multi–peripheral model [2].

At small momentum transfer, QCD perturbation theory is in general inapplicable, but one may choose to consider the scattering processes where the parton virtualities at the ends of the ladder are fixed to be large (for example, the scattering of two heavy quarkonium states [3]). However, even then the partons can still "diffuse" to small values of transverse momenta toward the center of the ladder (diffusion in the log of transverse momenta [1]), and at sufficiently high energies the perturbative approach inevitably breaks down [4]. This argument was formulated rigorously by A.H. Mueller [5], who showed that the operator product expansion (which provides the basis of the perturbative approach) breaks down at high energies. Another serious problem of the perturbative treatment has been made apparent by recent vigorous calculations of the next–to–leading corrections to the BFKL equation [6]. The NLO corrections appeared to be large, and drove the intercept of "BFKL Pomeron" significantly below the range of values suggested by phenomenology.

Perturbative expansion of the scattering amplitude is possible only in the presence of a sufficiently large scale. As was mentioned above, at very high energies the external scale, which determines the parton virtualities at the ends of the ladder, becomes progressively unimportant, and the perturbative expansion loses justification. Therefore it looks plausible that the Pomeron is a genuinely non–perturbative phenomenon [7], [8]. At present, non–perturbative phenomena can only be treated theoretically if they stem from relatively short distances, which requires the presence of a large scale. The main idea exploited in this letter is that such a scale exists in the QCD vacuum as a consequence of scale anomaly, and is related to the density of vacuum gluon fields due to semi–classical fluctuations; numerically, $M_0^2 \simeq 4 \div 6$ GeV^2 (see below). Because of the presence of this large scale, the perturbative expansion still makes sense; we are able also to evaluate explicitly the leading non–perturbative contribution due to the scale anomaly.

There are two facts that support the feasibility of such an approach. First, the success of QCD sum rules is based on the use of a few first terms in the operator product expansion, which can be justified only if a sufficiently large scale associated with the vacuum structure exists [9]. Second, the non–perturbative amplitude of low–energy dipole–dipole scattering was evaluated and found to be determined by the vacuum energy density, arising from the semi–classical fluctuations of gluon fields [10]. This latter example is encouraging, since the multi–peripheral model [2] relates the amplitude of high–energy scattering to the low–energy interactions of partons.

Basing on these ideas, we propose an extension of the BFKL program to the non-perturbative domain; the key ingredient of our approach is the breakdown of scale invariance in QCD, reflected in scale anomaly. The concept of scale anomaly is rather general and was formulated long time ago [11],[12],[13]; let us briefly recall its application to QCD. In the chiral limit of massless quarks, the Lagrangean of QCD is scale invariant on the classical, tree level. This invariance is however broken by renormalization, which introduces a dimensionful scale once the interactions are switched on. This "dimensional transmutation" phenomenon is fundamental for the understanding of scale dependence of the strong coupling constant [14], which is the basis of all applications of perturbative QCD. On the formal level, the breakdown of scale invariance in the theory is reflected by the non–conservation of scale current, and thus in the non–zero trace of the energy–momentum tensor $\theta^{\mu\nu}$ [15]. Scale anomaly leads to a set of powerful low-energy theorems for the correlation functions of gluon currents in the scalar channel

[9].

The starting point of the approach that we propose in this letter is the following: among the higher order, $O(\alpha_S^2)$ ($\alpha_S = g^2/4\pi$) , corrections to the BFKL kernel we isolate a particular class of diagrams which include the propagation of two gluons in the scalar color singlet channel $J^{PC} = 0^{++}$ (see Fig. 1-b). We will show that, as a consequence of scale anomaly, these, apparently $O(\alpha_S^2)$, contributions become the *dominant* ones, $O(\alpha_S^0)$.

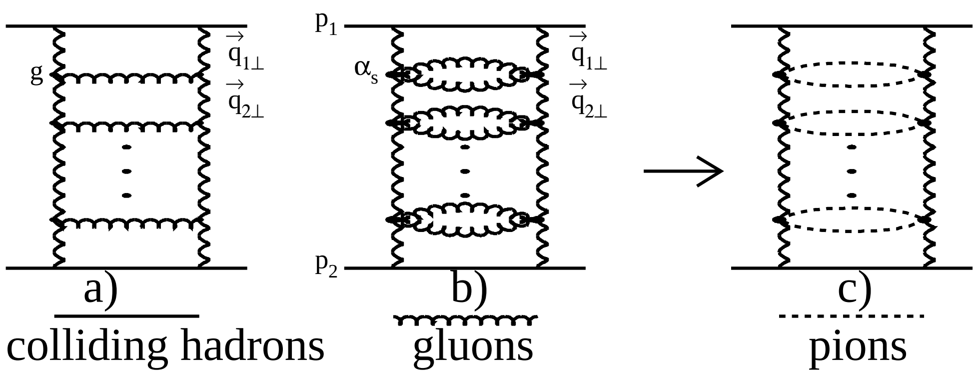

Figure 1: *Multi-peripheral (ladder) diagrams contributing to the leading–order BFKL (a) and "soft" (b) and (c) Pomeron structure*

Indeed, let us consider the contribution of Fig. 1-b, which is one of the numerous corrections of the next–to–leading order to the BFKL Pomeron. In perturbation theory, such corrections are $\propto \alpha_S^2$; however, we note that if the two produced gluons in Fig.1-b are in the scalar and colorless state, the vertex of their production, generated the four–gluon coupling in the QCD Lagrangean, is $\sim \alpha_s F^{\mu\nu\ a} F^a_{\mu\nu}$. We observe that this vertex is therefore proportional to the trace of the QCD energy–momentum tensor (θ^μ_μ) in the chiral limit of massless quarks:

$$\theta^\mu_\mu = \frac{\beta(g)}{2g} F^{a\alpha\beta} F^a_{\alpha\beta} \simeq -\frac{bg^2}{32\pi^2} F^{a\alpha\beta} F^a_{\alpha\beta}; \tag{1}$$

note that as a consequence of decoupling theorem [16] the β function in Eq. (1) does not contain the contribution of heavy quarks (*i.e.* $b = \frac{1}{3}(11N - 2N_f) = 9$).

The entire contribution of Fig.1-b therefore appears proportional to the correlator of the QCD energy–momentum tensor. Let us now consider the spectral representation for this correlator:

$$\Pi(q^2) = i \int d^4x\ e^{iqx}\ \langle 0|\mathrm{T}\left\{\theta^\mu_\mu(x)\theta^\nu_\nu(0)\right\}|0\rangle = \int d\sigma^2 \frac{\rho_\theta(\sigma^2)}{\sigma^2 - q^2 - i\epsilon}, \tag{2}$$

with the spectral density defined by

$$\rho_\theta(k^2) = \sum_n (2\pi)^3 \delta^4(p_n - k) |\langle n|\theta^\mu_\mu|0\rangle|^2, \tag{3}$$

where the phase-space integral is understood. In lowest–order perturbation theory, the spectral density (3) is given by the contribution of two-gluon states; the calculation for $SU(N)$ color gives

$$\rho_\theta^{\rm pt}(q^2) \quad = \quad \left(\frac{bg^2}{32\pi^2}\right)^2 \frac{N_c^2 - 1}{4\pi^2}\ q^4. \tag{4}$$

However, at small invariant masses, perturbation theory inevitably breaks down; an important theorem [9] for this correlator states that as a consequence of broken scale invariance of QCD,

$$\Pi(0) = -4\ \langle 0|\theta^\mu_\mu(0)|0\rangle. \tag{5}$$

Since this theorem, as will become clear soon, is a corner–stone of our approach, let us briefly recall its proof [9]. It is based on the fact that the expectation value of any operator O of canonical dimension d ($d = 4$ for θ^μ_μ) can be written down as

$$\langle O \rangle \sim \left[M_0\ exp\left(-\frac{8\pi^2}{bg_0^2}\right)\right]^d, \tag{6}$$

where $g_0 \equiv g(M_0)$, and M_0 is the renormalization scale. On the other hand, the dependence of QCD Lagrangean on the coupling is $(-1/4g_0^2)\ \tilde{F}^{a\alpha\beta}\tilde{F}^a_{\alpha\beta}$, where $\tilde{F} \equiv gF$ is the rescaled gluon field. By writing down the expectation value of the operator O in the form of the functional integral, and by differentiating this expression with respect to $1/4g_0^2$, one can therefore generate correlation functions of the operators O and $\tilde{F}^2$. Differentiating once, one gets

$$i \int dx\ \langle\ \mathrm{T}\left\{O(x)\ \tilde{F}^2(0)\right\}\rangle \equiv -\frac{d}{d(1/4g^2)}\langle O\rangle. \tag{7}$$

Combining (7) and (6), and choosing $O(x) = \theta^\mu_\mu(x)$, furnishes the proof of the theorem (5).

Note that the r.h.s. of Eq. (5) is divergent even in perturbation theory, and should therefore be regularized by subtracting the perturbative part. The vacuum expectation value of the θ^μ_μ operator then measures the energy density of non-perturbative fluctuations in QCD vacuum, and the low-energy theorem (5) implies a sum rule for the spectral density:

$$\int \frac{d\sigma^2}{\sigma^2}[\rho_\theta^{\rm phys}(\sigma^2) - \rho_\theta^{\rm pt}(\sigma^2)] = -4\ \langle 0|\theta^\mu_\mu(0)|0\rangle = -16\ \epsilon_{\rm vac} \neq 0, \tag{8}$$

where the estimate for the vacuum energy density extracted from the sum rule analysis gives $\epsilon_{\rm vac} \simeq -(0.24\ \text{GeV})^4$ [17]. In addition, another sum rule [18, 17],

$$\int d\sigma^2 \rho_\theta^{\rm phys}(\sigma^2) = \int d\sigma^2 \rho_\theta^{\rm pt}(\sigma^2) \tag{9}$$

is implied by the quark–hadron duality. Since the physical spectral density, $\rho_\theta^{\rm phys}$, should approach the perturbative one, $\rho_\theta^{\rm pt}$, at high σ^2, the integral in Eq. (8) is convergent.

According to (1) and (3), the l.h.s. of Eq.(8) is apparently $O(g^4)$; however it is easy to see that this is not so by looking at the r.h.s. of this equation, which is renormalization group invariant, and does not depend on the coupling constant. This means that the l.h.s. must also be $O(g^0)$. Let us illustrate this formal argument by considering the spectral density (3) at small invariant mass [19]. Small invariant masses imply small relative momenta for the produced particles, and at small momenta an accurate description of QCD is given by an effective chiral Lagrangean

$$\mathcal{L} = \frac{f_\pi^2}{4}\ {\rm tr}\ \partial_\mu U \partial^\mu U^\dagger\ +\ \frac{1}{4}\ m_\pi^2 f_\pi^2\ {\rm tr}\left(U + U^\dagger\right), \tag{10}$$

where $U = \exp\left(2i\pi/f_\pi\right)$, $\pi \equiv \pi^a T^a$ and T^a are the $SU(2)$ generators normalized by tr $T^a T^b = \frac{1}{2}\delta^{ab}$. The trace of the energy–momentum tensor for this Lagrangian is (see, e.g., [10])

$$\theta_\mu^\mu = -2\ \frac{f_\pi^2}{4}\ {\rm tr}\ \partial_\mu U \partial^\mu U^\dagger\ -\ m_\pi^2 f_\pi^2\ {\rm tr}\left(U + U^\dagger\right). \tag{11}$$

Expanding this expression (11) in powers of the pion field, one obtains, to the lowest order,

$$\theta_\mu^\mu = -\partial_\mu \pi^a \partial^\mu \pi^a + 2m_\pi^2 \pi^a \pi^a + \cdots, \tag{12}$$

and this leads to an elegant result [19] in the chiral limit of vanishing pion mass:

$$\langle \pi^+ \pi^- | \theta_\mu^\mu | 0 \rangle = q^2\ . \tag{13}$$

This result for the coupling of the operator θ_μ^μ to two pions can be immediately generalized for any (even) number of pions using Eq. (11). The expression (13) is manifestly $\sim O(g^0)$, and shows that the spectral density of the scalar gluon operator $\sim g^2 F^2$ is independent of the coupling constant g as a consequence of scale anomaly. While we have used an effective chiral Lagrangean to illustrate how the dependence on the coupling constant gets "eaten" by the scale anomaly, this phenomenon is very general and does not depend on the specific model for the spectral density. One way of understanding the disappearance of the coupling constant in the spectral density of the $g^2 F^2$ operator is to assume that the non-perturbative QCD vacuum is dominated by the semi–classical fluctuations of the gluon field. Since the strength of the classical gluon field is inversely proportional to the coupling, $F \sim 1/g$, the quark zero modes, and the spectral density of their pionic excitations, appear independent of the coupling constant.

Armed with this knowledge, we are ready to see that the contribution to the next–to–leading order BFKL kernel that describes the production of two gluons in the color singlet, scalar state, which is formally $\sim O(g^4)$, as a consequence of scale anomaly can become the leading one, $\sim O(g^0)$. (Of course, the perturbative part of this contribution is still $\sim O(g^4)$ and has been taken into account in the next-to-leading order BFKL Pomeron). Therefore, we want to build a multi-peripheral model for the "soft" Pomeron in which hadrons are produced (mostly two pions, see Refs.[10], [20]) due to exchange of two gluons in the t–channel (see Fig.1-c). The only dimensional scale in this approach appears in ρ^{phys} and can be estimated directly from sum rules of Eq. (8). It turns out that the characteristic mass (M_0^2) in Eq. (8)

is rather large [10], [20] $M_0^2 \approx 4\,\mathrm{GeV}^2$ (the original analysis of [9], [21] yielded even somewhat bigger value $M_0^2 \approx 6\,\mathrm{GeV}^2$). This is the largest scale which exists in non–perturbative QCD [9], [22]. In the framework of the instanton approach, the large magnitude of M_0 was shown to be a consequence of strong color field inside the instanton [22].

This value determines the scale of all dimensional parameters of the Pomeron trajectory as well as the typical transverse momentum of produced particle in the Pomeron. It is interesting to notice that the experimental value for the slope $\alpha'_P(0)$ of the Pomeron trajectory (in the standard notation, $\alpha_P(t) = 1 + \Delta + \alpha'_P(0)\,t$) $\alpha'_P(0) = 0.25\,\mathrm{GeV}^{-2}$[23] is very close to $1/M_0^2$.

We start our calculation with diagrams of Fig.1-b in the leading log s approximation of pQCD where we sum only contributions of the order of $(\alpha_S \ln s)^n$. In this approximation the propagators of the t-channel gluons can be written in a simple form [24]

$$G_{\mu\nu}(q_i^2) \;=\; \frac{g_{\mu\nu}}{q_i^2} \;=\; \frac{1}{q_{i,\perp}^2} \times \frac{2\,q_{i,\perp,\mu}\,q_{i,\perp,\nu}}{\alpha_i\,\beta_i\,s} \;+\; O\left(\frac{1}{s}\right)\;, \tag{14}$$

where we use the Sudakov decomposition for momenta q_i along the momenta of colliding particles (p_1 and p_2 in Fig.1-b), namely,

$$q_{i,\mu} \;=\; \alpha_i\,p_{1,\mu} \;+\; \beta_i\,p_{2,\mu} \;+\; q_{i,\perp,\mu}\;; \tag{15}$$

Eq. (14) corresponds to Weizsäcker–Williams approximation for the gluon field of a fast–moving hadron.

The ladder diagram of Fig.1-b for emission of n-pairs is equal to

$$\begin{aligned}\sigma_n(Q^2) \;&=\; \alpha_S^2 \int \Gamma_\mu \Gamma_\nu \prod_{i=1}^{i=n+1} \frac{s\;d\alpha_i d\beta_i d^2 q_{i,\perp}}{2(2\pi)^3}\,\delta(\alpha_i\beta_{i+1}s - M_{i,\perp}^2)\,\Phi_i \\ &\cdot\;\; \Gamma_{\mu_i,\mu_{i+1}}\Gamma_{\nu_i,\nu_{i+1}} G_{\mu_i,\mu_{i+1}}(q_i^2) G_{\nu_i,\nu_{i+1}}((Q-q_i)^2)\,\Gamma_{\mu_i+1}\Gamma_{\nu_i+1}\;\;,\end{aligned} \tag{16}$$

where $\sigma(Q^2=0)$ is the total cross section of n-pairs production (σ_n) and Φ_i is the phase space factor for two identical particles with total mass M_i which is equal to

$$\Phi_i \;=\; \frac{1}{2}\int \frac{d^3k_1}{(2\pi)^3 2\omega_1 2\omega_2}\,\delta(\omega_1+\omega_2-M_i) \;=\; \frac{1}{32\pi^2}\;. \tag{17}$$

In Eq. (16) $\Gamma_{\mu_i,\mu_{i+1}}$ is the vertex of gluon pair production. It is easy to calculate that it is equal to

$$q_{i,\perp,\mu_i}\,q_{i,\perp,\mu_{i+1}}\,\Gamma_{\mu_i,\mu_{i+1}} \;=\; 3\,\vec{q}_{i,\perp}\cdot\vec{q}_{i+1,\perp}\;, \tag{18}$$

after projecting on the colorless state with $J^{PC} = 0^{++}$.

Using the kinematic relation $\alpha_i\,\beta_{i+1}\,s = M_i^2 + k_{i,\perp}^2$, where $\vec{k}_{i,\perp} = \vec{q}_{i,\perp} - \vec{q}_{i+1,\perp}$ and performing integration over α_i explicitly, we can rewrite Eq. (16) in the simple form

$$\sigma_n(Q^2) \;=\; \alpha_S^2\,\frac{(\ln s)^n}{n!}\prod_{i=1}^{i=n+1}\;\frac{4\cdot 9\cdot\alpha_S^2}{32\pi^2}\,\frac{(\vec{Q}_\perp - \vec{q}_{i,\perp})\cdot(\vec{Q}_\perp - \vec{q}_{i+1,\perp})}{q_{i,\perp}^2\,(\vec{Q}-\vec{q}_i)_\perp^2}\;\frac{dM_i^2}{\left(M_i^2 + k_{i,\perp}^2\right)^2}\;. \tag{19}$$

For forward scattering $Q_\perp^2 = 0$ and Eq. (19) leads to power–like behavior of the total cross section:

$$\sigma_{tot} = \sum_{n=0}^{\infty} \sigma_n \ = \sigma^{BORN} s^{\Delta} \ , \tag{20}$$

where

$$\Delta \ = \ \frac{\alpha_S^2 \ 18}{32\pi^2} \int \frac{dk_\perp^2 \ dM^2}{\left(M^2 \ + \ k_\perp^2\right)^2} \ . \tag{21}$$

and σ^{BORN} is the cross section due to two gluon exchange

$$\sigma^{BORN} \ = \ \alpha_S^2 \int d^2q \Gamma_\mu \Gamma_\nu G_{\mu,\mu_1}(q_\perp^2) \ G_{\nu,\nu_1}(q_\perp^2) \ \Gamma_{\mu_1}\Gamma_{\nu_1} \ . \tag{22}$$

Eq. (21) can be easily rewritten through the perturbative spectral density ρ_θ^{pQCD} that was evaluated above (see Eq. (4)):

$$\Delta \ = \frac{\pi^2}{2} \times \left(\frac{8\pi}{b}\right)^2 \times \frac{18}{32\pi^2} \int \frac{dM^2}{M^6} \, \rho_\theta^{pQCD}(M^2) \ , \tag{23}$$

where $\alpha_S(M^2) = 4\pi/(b \ln(M^2/\Lambda^2))$.

Our main idea is to separate non–perturbative and perturbative contributions to the spectral density of the scalar gluon operator by replacing $\rho_\theta^{pQCD}(M^2)$ with $(\rho_\theta^{phys}(M^2) \ - \ \rho_\theta^{pQCD}(M^2)) + \rho_\theta^{pQCD}(M^2)$. The purely perturbative contribution is of the order of $O(g^4)$, and has been evaluated before [6]. For the non–perturbative contribution, in which we are interested here, we have

$$\Delta \ = \frac{\pi^2}{2} \times \left(\frac{8\pi}{b}\right)^2 \times \frac{18}{32\pi^2} \int \frac{dM^2}{M^6} \left(\rho_\theta^{phys}(M^2) \ - \ \rho_\theta^{pQCD}(M^2)\right) \ . \tag{24}$$

To estimate the integral in Eq. (24) we will use the chiral approach to ρ_θ^{phys} described above, namely,

$$\rho_\theta^{phys}(M^2) \ = \ \frac{3}{32\pi^2} \, M^4 \ , \tag{25}$$

which corresponds to diagram of Fig. 1-c [1].

It is instructive to establish a qualitative relation between the matching parameter M_0 and the energy density of QCD vacuum using the spectral density (25) and the sum rule (8). Since perturbative spectral density $\rho_\theta^{pQCD}(M^2)$ at moderate M is much smaller than $\rho_\theta^{phys}(M^2)$, Eqs (25) and (8) lead to the following approximate relation [21]:

$$M_0^2 \ \ \simeq \ \ 32\pi \left\{ \frac{|\epsilon_{\rm vac}|}{N_f^2 - 1} \right\}^{\frac{1}{2}} , \tag{26}$$

which shows that the matching scale M_0 is directly determined by the energy density of the vacuum. Since $\epsilon_{\rm vac} \sim N_c^2$, the magnitude of M_0 is proportional to N_c^2/N_f^2.

[1] Of course, the chiral approach cannot be extrapolated up to $M_0 \simeq 2$ GeV; at large M the spectral density (25) should be corrected by a phenomenological form-factor expressed in terms of experimental $\pi\pi$ phase shifts [10]; however, since our sensitivity to the large M region is only logarithmic (see (24)), in this paper we will use the simplified ansatz (25).

Collecting all numerical factors and substituting $b = 9$, we obtain

$$\Delta \;=\; \frac{1}{48}\;\ln\frac{M_0^2}{4m_\pi^2}\;. \tag{27}$$

Let us discuss the result of these simple calculations:

1. Eq. (20) and Eq. (27) say that our approach leads to the exchange of the Pomeron with the intercept $\alpha(0) = 1 + \Delta > 1$. To the best of our knowledge, this is the only approach in the framework of QCD which leads to a "soft" Pomeron;

2. Eq. (20) is a consequence of a direct generalization of the BFKL approach to the non–perturbative domain. It should be recalled that the only theory where the Pomeron naturally appears is the BFKL Pomeron in $2+1$ dimensional QCD [25];

3. Eq. (27) gives $\Delta = 0.082$ for $M_0^2 = 4\,\mathrm{GeV}^2$ [10] in good agreement with the phenomenological intercept of the "soft" Pomeron, $\Delta = 0.08$ [23]; it should be noted however that the precise value of the matching scale M_0^2 as extracted from the low–energy theorem (8) depends somewhat on detailed form of the spectral density, and can vary within the range of $M_0^2 = 4 \div 6\,\mathrm{GeV}^2$ [9], [10]. Fortunately, the dependence of Eq. (27) on M_0 is only logarithmic, and varying it in this range leads to

$$\Delta = 0.08 \div 0.1. \tag{28}$$

4. As we have already stressed, in our approach the only dimensionful parameter is M_0^2; its large value implies the dominance of rather short distances in the "soft" Pomeron structure. This fact is in agreement with a number of experimental and phenomenological observations:

   - The value of the slope for the "soft" Pomeron trajectory $\alpha'_P(0) = 0.25\,\mathrm{GeV}^{-2} \ll \alpha'_R(0) = 1\,\mathrm{GeV}^{-2}$, where α'_R is the slope of the Reggeon trajectory;

   - The experimental slope of the diffraction production of the hadron system with large mass is approximately two times smaller the slope for the elastic scattering. It means that the proper size of the triple Pomeron vertex is rather small. For our Pomeron it should be on the order of $1/M_0^2 \approx 0.25\,\mathrm{GeV}^{-2} \ll B_{el} = 10\,\mathrm{GeV}^{-2}$;

   - The HERA data [26] on diffractive J/Ψ production in DIS show that the t - slope for elastic diffractive dissociation ($\gamma^* + p \to J/\Psi + p$) is larger than the t - slope for the inelastic one ($\gamma^* + p \to J/\Psi + X$, where X is a high–mass hadronic system). This shows the existence of two different scales in the proton, one of which is determined by its size, and another one by the correlation length $\sim 1/M_0$ of the gluon field inside.

5. Non–trivial azimuthal dependence observed recently in diffractive production of scalar mesons [27] can be explained [28] if one adopts the idea that the effective coupling of the Pomeron to mesons is dictated by scale anomaly.

The disappearance of the dependence on the coupling constant, which is the central point of our approach, may seem puzzling. However let us mention again that this result can be easily understood if we recall the interpretation of the non–zero v.e.v. of the trace of the energy–momentum tensor as being due to the semi–classical fluctuations of gluon fields. Since the strength of the classical gluon field is $F^2 \sim 1/\alpha_S$, quark zero modes, and their pionic excitations, appear independent of the coupling, $\sim O(\alpha_S^0)$. We therefore envision the Pomeron as a $t-$channel exchange of two gluons, which scatter off semi–classical fluctuations of vacuum gluon fields; this scattering is accompanied by the excitation of quark zero modes in the vacuum, resulting in the production of pions. Amazingly similar picture of the "soft" Pomeron was anticipated by Bjorken [32]. These effects also manifest themselves in the low–energy scattering of heavy quarkonia; the magnitude of the scattering amplitude was found [10] to be determined by the energy density of the non–perturbative QCD vacuum.

Let us discuss the dependence of our result on the numbers of colors, N_c, and flavors, N_f. Two limits are of theoretical interest: *i)* $N_c \to \infty$, N_f, $g^2\ N_c$ fixed; *ii)* $N_c \to \infty$, N_f/N_c, $g^2\ N_c$ fixed. The case *i)* corresponds to the large N_c limit proposed by 't Hooft [29], while *ii)* is the basis of "topological expansion" suggested by Veneziano [8].

Since the number of Goldstone bosons contributing to the non–perturbative spectral density (25) for spontaneously broken $SU_L(N_f) \times SU_R(N_f)$ is equal to $N_f^2 - 1$, it is evident from Eq. (24) that $\Delta \sim N_f^2/N_c^2$ (note that Eq. (24) contains $b = 1/3(11N_c - 2N_f)$ in the denumerator). A simple graphic illustration of this dependence is given in Fig.2. Therefore, our approach may

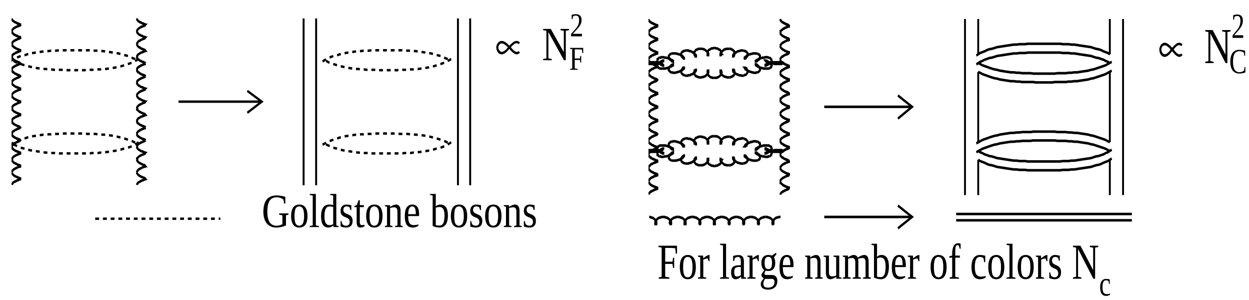

Fig. 2-a

Fig.2-b

Figure 2: *A simple illustration of the appearance of the N_f^2/N_c^2 factor in the Pomeron intercept.*

be considered as a realization of general ideas, proposed long time ago by Veneziano [8], that the "soft" Pomeron should be found keeping N_f^2/N_c^2 fixed.

Let us discuss now the large N_c limit *i)*, which corresponds to pure gluodynamics. Naively, since Δ was found proportional to N_f^2/N_c^2, one may conclude that in this limit Δ vanishes, and the cross section does not grow with energy. This conclusion is, however, immature. Indeed,

the physical spectrum in the scalar channel in gluodynamics contains a scalar glueball [2]; its couplings to mesons are suppressed by $1/N_c$ (see, e.g., [30]), so it should be very narrow. Therefore, the spectral density Eq. (25) should be replaced by

$$\rho_\theta^{phys}(M^2) \; = \; R\, M_R^6 \; \delta(M^2 - M_R^2) + pert.\ contribution \tag{29}$$

where M_R is the scalar glueball mass, and R is its residue; the factor M_R^6 is introduced to make R dimensionless. Using Eq. (29) in the sum rule (8), we get a simple relation

$$R = 16\ \frac{|\epsilon_{\rm vac}|}{M_R^4}. \tag{30}$$

With Eq. (29) and Eq. (30), Eq. (24) becomes

$$\Delta = \frac{288\pi^2}{b^2}\ \frac{|\epsilon_{\rm vac}|}{M_R^4}; \tag{31}$$

since $M_R \sim N_c^0$, $\epsilon_{\rm vac} \sim N_c^2$, and $b \sim N_c$, Eq. (31) is well–defined in the large N_c limit.

One could try to estimate the value of Δ given by Eq. (31) for $N_c = 3$; this requires the knowledge of the mass of the scalar glueball in pure gluodynamics. Recent lattice result [31] gives $M_R \simeq 1.65$ GeV; assuming that the main contribution to the energy density of the vacuum is due to gluons and using, as before, $\epsilon_{\rm vac} \simeq (0.24\ {\rm GeV})^4$, $b = 11N_c/3 = 11$, we get the value

$$\Delta_{gluodynamics} \simeq 0.01, \tag{32}$$

which is significantly smaller than our result (28) for the world with light quarks. This indicates that the presence of light quarks in the theory leads to a much faster growth of the cross section with energy.

The key question is whether one can prove the theoretical self-consistency of our approach. Indeed, our classification of the contributions to the scattering amplitude is still based on the expansion in powers of α_S, in which we have isolated the term $\sim O(\alpha_S^0)$ emerging as a consequence of scale anomaly. This term is the leading one only if the coupling constant α_S is sufficiently small. The magnitude of the coupling depends on the renormalization scale M_0. Since this dimensionful scale extracted from the sum rule analysis is large, $M_0 = 4 \div 6\ {\rm GeV}^2$, the coupling constant indeed appears to be small, $\alpha_S(M_0^2) \ll 1$. This fact insures that perturbative corrections to the kernel are smaller than the leading, $\sim O(\alpha_S^0)$, term, and should be taken into account in the framework of conventional BFKL approach. Our approach yields a natural rapidity scale ($Y_0 = ln(M_0^2/4m_\pi^2) \simeq 2 \div 3$) for BFKL kernel above which the perturbative approach can be applied. It is interesting to note that for $Y \geq Y_0$ the next–to–leading order corrections are well under control [33]; however the interplay between "soft" and "hard" physics still has to be understood.

We therefore believe that our proposal can lead to a systematic theoretical approach to the Pomeron in QCD. Of course, one cannot exclude *a priori* a different view of the Pomeron

[2] We thank S. Nussinov and E. Shuryak for stressing the rôle of the scalar glueball for our approach in the case of pure gluodynamics.

structure as coming from large distances, $R \gg 1/M_0$. However, experimental data (see point 4 of our discussion above) suggest that the phenomenological Pomeron indeed originates at small distances $R \sim \sqrt{\alpha'_P(0)} \sim 1/M_0$.

Let us discuss the relation of our approach to other existing approaches to soft scattering. Very similar ideas of the dominance of semi–classical vacuum gluon fields were developed in Refs. [34]. Our approach is complementary to these ideas, giving a natural explanation of the energy behavior of the soft scattering amplitude, which previously had to be taken phenomenologically[3]. Let is note also that the correlation length of gluon fields, which was taken from the lattice QCD calculations in Refs [34], in our approach appears only implicitly and is determined from the analysis of low–energy theorems.

The dominance of classical gluon field configurations in high–energy collisions is the key idea of the approach proposed by McLerran and Venugopalan [36] and developed in Refs. [37]. In this approach, the rôle of dimensionful parameter is played by the density of color charges in the transverse plane, rather than by the vacuum energy density. In our opinion, this is a plausible assumption at very high energies and/or for sufficiently heavy nuclei for low partial amplitudes (central region in the impact parameter plane). Since we focus our attention on the behavior of the total cross section, which is determined by large distances in the impact parameter space, and therefore small density of the color charge, the relevance of the scale M_0^2 associated with the vacuum field strength should not be surprising. We feel that the approach of [36], [37] can describe the inclusive cross section, while ours is suited for the description of the total cross section. Indeed, the multiplicity associated with our multi–peripheral ladder $\sim 2\ \alpha(0)\ ln\ s$ is rather small compared to the expectations of [36], [37]. It would be extremely interesting to understand better the relationship between the two approaches.

Acknowledgements

We thank Ian Balitsky, James Bjorken, Wilfried Buchmüller, Günter Dosch, John Ellis, Hirotsugu Fujii, Errol Gotsman, Bob Jaffe, Alexei Kaidalov, T.D. Lee, Uri Maor, Larry McLerran, Al Mueller, Shmuel Nussinov, Rob Pisarski, Misha Ryskin, Edward Shuryak, Chung–I Tan, Larry Trueman and Raju Venugopalan for very fruitful discussions of problems related to this work.

The work of D.K. was supported by the US Department of Energy (Contract # DE-AC02-98CH10886) and RIKEN. The research of E.L. was supported in part by the Israel Science Foundation, founded by the Israeli Academy of Science and Humanities, and BSF # 9800276.

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